Trigonometric Formula Sheet

De nition of the Trig Functions

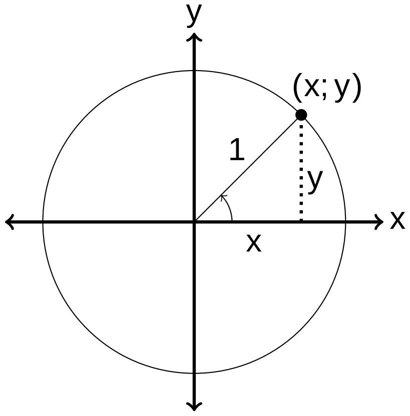
Right Triangle De nition

Assume that:

$0 < \quad < \frac{}{2}$ or 0

Unit Circle De nition

Assume can be any angle.



sin = $\frac{opp}{}$ csc = $\frac{hyp}{}$. y

Identities and Formulas

Tangent and Cotangent Identities

$\tan = \frac{\sin}{\cos} \qquad \cot = \frac{\cos}{\sin}$

Reciprocal Identities

$\sin = \frac{1}{\csc} \qquad \csc = \frac{1}{\sin}$

$\cos = \frac{1}{\sec} \qquad \sec = \frac{1}{\cos}$

$\tan = \frac{1}{\cot} \qquad \cot = \frac{1}{\tan}$

Pythagorean Identities

$\sin^2 + \cos^2 = 1$

$\tan^2 + 1 = \sec^2$

$1 + \cot^2 = \csc^2$

Even and Odd Formulas

$\sin(\ ) = \ \sin \qquad \csc(\ ) = \ \csc$

$\cos(\ ) = \cos \qquad \sec(\ ) = \sec$

$\tan(\ ) = \ \tan \qquad \cot(\ ) = \ \cot$

Periodic Formulas

If n is an integer

$\sin(\ + 2\ n\ ) = \sin \qquad \csc(\ + 2\ n\ ) = \csc$

$\cos(\ + 2\ n\ ) = \cos \qquad \sec(\ + 2\ n\ ) = \sec$

$\tan(\ +\ n\ ) = \tan \qquad \cot(\ +\ n\ ) = \cot$

Double Angle Formulas

$\sin(2\ ) = 2\sin\ \cos$

$\cos(2\ ) = \cos^2\ \ \sin^2$

$= 2\cos^2\ \ 1$

$= 1\ \ 2\sin^2$

$\tan(2\ ) = \frac{2\tan}{1\ \ \tan^2}$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then:

$\frac{}{180} = \frac{t}{x} \quad) \quad t = \frac{x}{180}$ and $x = \frac{180\,t}{}$

Half Angle Formulas

$\sin = \ r\sqrt{\frac{1\ \ \cos(2\ )}{2}}$

$\cos = \ r\sqrt{\frac{1 + \cos(2\ )}{2}}$

$\tan = \ s\sqrt{\frac{1\ \ \cos(2\ )}{1 + \cos(2\ )}}$

Sum and Di erence Formulas

$\sin(\ \ ) = \sin\ \cos\ \ \cos\ \sin$

$\cos(\ \ ) = \cos\ \cos\ \ \sin\ \sin$

$\tan(\ \ ) = \frac{\tan\ \ \tan}{1\ \ \tan\ \tan}$

Product to Sum Formulas

$\sin\ \sin = \frac{1}{2}[\cos(\ \ )\ \ \cos(\ +\ )]$

$\cos\ \cos = \frac{1}{2}[\cos(\ \ ) + \cos(\ +\ )]$

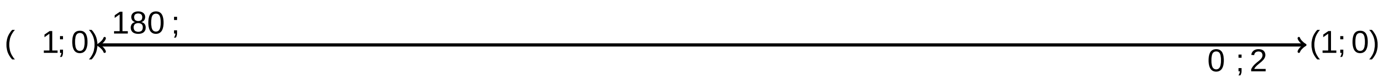
$\sin\ \cos = \frac{1}{2}[\sin(\ +\ ) + \sin(\ \ )]$

$\cos\ \sin = \frac{1}{2}[\sin(\ +\ )\ \ \sin(\ \ )]$

Sum to Product Formulas

$\sin\ + \sin\ = 2\sin\ \frac{+}{2}$

Unit Circle



Inverse Trig Functions

De nition

= sin ^{1}(x) is equivalent to x = sin

= cos ^{1}(x) is equivalent to x = cos

= tan ^{1}(x) is equivalent to x = tan

Inverse Properties

These properties hold for x in the domain and in the range

sin(sin ^{1}(x)) = x	sin ^{1}(sin( )) =
cos(cos ^{1}(x)) = x	cos ^{1}(cos( )) =
tan(tan ^{1}(x)) = x	tan ^{1}(tan( )) =

Domain and Range

Function	Domain	Range
= sin ^{1}(x)	1 x 1	$\overline{2}$ $\overline{2}$
= cos ^{1}(x)	1 x 1	0
= tan ^{1}(x)	1 x 1	$\overline{2}$ < < $\overline{2}$

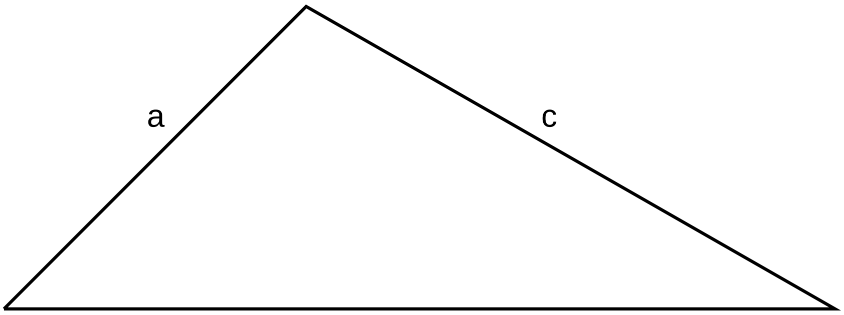
Other Notations

sin ^{1}(x) = arcsin(x)

cos ^{1}(x) = arccos(x)

tan ^{1}(x) = arctan(x)

Law of Sines, Cosines, and Tangents



Law of Sines

$$\frac{\sin}{a} = \frac{\sin}{b} = \frac{\sin}{c}$$

Law of Cosines

$a^2 = b^2 + c^2$ 2bccos

$b^2 = a^2 + c^2$ 2accos

$c^2 = a^2 + b^2$ 2abcos

Law of Tangents

$$\frac{a \quad b}{a + b} = \frac{\tan \frac{1}{2}(\quad)}{\tan \frac{1}{2}(\; + \;)}$$

$$\frac{b \quad c}{b + c} = \frac{\tan \frac{1}{2}(\quad)}{\tan \frac{1}{2}(\; + \;)}$$

$$\frac{a \quad c}{a + c} = \frac{\tan \frac{1}{2}(\quad)}{\tan \frac{1}{2}(\; + \;)}$$

Complex Numbers

$$i=\sqrt{-1} \qquad i^2=-1 \qquad i^3=-i \qquad i^4=1$$

$\sqrt{-a}=i\sqrt{a};\ a\geq 0$

$(a+bi)(a-bi)=a^2+b^2$

$(a+bi)+(c+di)=a+c+(b+d)i$

$|a+bi|=\sqrt{a^2+b^2}$ Complex Modulus

$(a+bi)-(c+di)=a-c+(b-d)i$

$\overline{(a+bi)}=a-bi$ Complex Conjugate

$(a+bi)(c+di)=ac-bd+(ad+bc)i$

$\overline{(a+bi)}(a+bi)=|a+bi|^2$

DeMoivre's Theorem

Let $z=r(\cos\theta+i\sin\theta)$, and let n be a positive integer.
Then:

$$z^n=r^n(\cos n\theta+i\sin n\theta).$$

Example: Let $z=1-i$, find z^6.

Finding the nth roots of a number using DeMoivre's Theorem

Example: Find all the complex fourth roots of 4. That is, find all the complex solutions of $x^4 = 4$.

We are asked to find all complex fourth roots of 4.
These are all the solutions (including the complex values) of the equation $x^4 = 4$.

For any positive integer n, a nonzero complex number z has exactly n distinct nth roots.
More specifically, if z is written in the trigonometric form $r(\cos\theta + i\sin\theta)$, the nth roots of z are given by the following formula.

$$(\ast)\quad r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{360k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{360k}{n}\right)\right);\quad \text{for } k = 0; 1; 2; \ldots; n-1.$$

Remember from the previous example we need to write 4 in trigonometric form by using:

$$r = \sqrt{(a)^2 + (b)^2} \quad \text{and} \quad \theta = \arg(z) = \tan^{-1}\left(\frac{b}{a}\right).$$

So we have the complex number $a + ib = 4 + i0$.

Therefore $a = 4$ and $b = 0$

So $r = \sqrt{(4)^2 + (0)^2} = 4$ and

$\theta = \arg(z) = \tan^{-1}\left(\frac{0}{4}\right) = 0$

Finally our trigonometric form is $4 = 4(\cos 0 + i\sin 0)$

Using the formula $(\ast)$ above with $n = 4$, we can find the fourth roots of $4(\cos 0 + i\sin 0)$

For $k = 0$; $4^{\frac{1}{4}}\left(\cos\left(\frac{0}{4} + \frac{360}{}\right.\right.$

Formulas for the Conic Sections

Circle

$$\text{StandardForm}: (x \quad h)^2 + (y \quad k)^2 = r^2$$

Where $(h;k) =$ center and r center and r :and

More Conic Sections

Hyperbola

Standard Form for Horizontal Transverse Axis :

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Standard Form for Vertical Transverse Axis :

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Where $(h;k)$ = center

a=distance between center and either vertex

Foci can be found by using $b^2 = c^2 - a^2$

Where c is the distance between

center and either focus. $(b > 0)$

Parabola

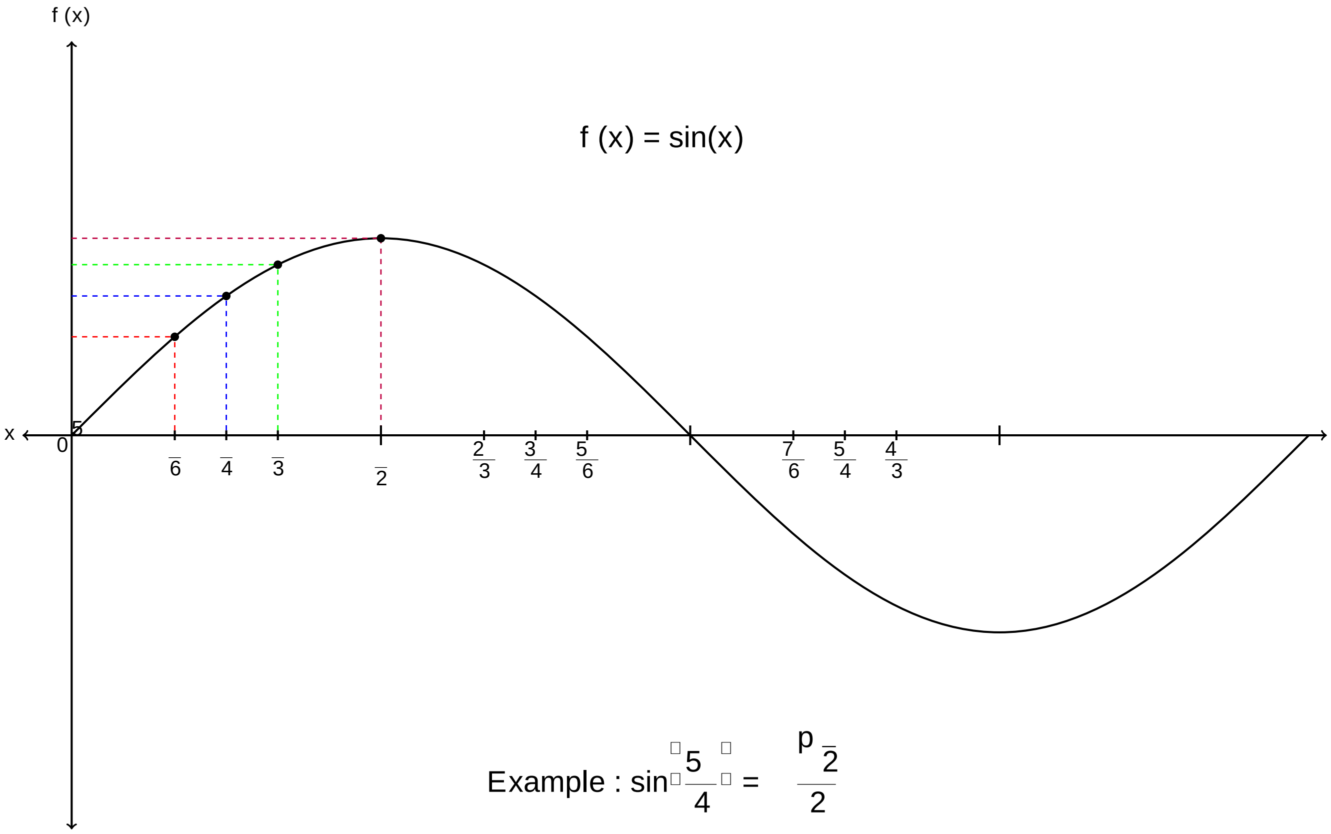
Vertical axis: $y = a(x-h)^2 + k$

Horizontal axis: $x = a(y-k)^2 + h$

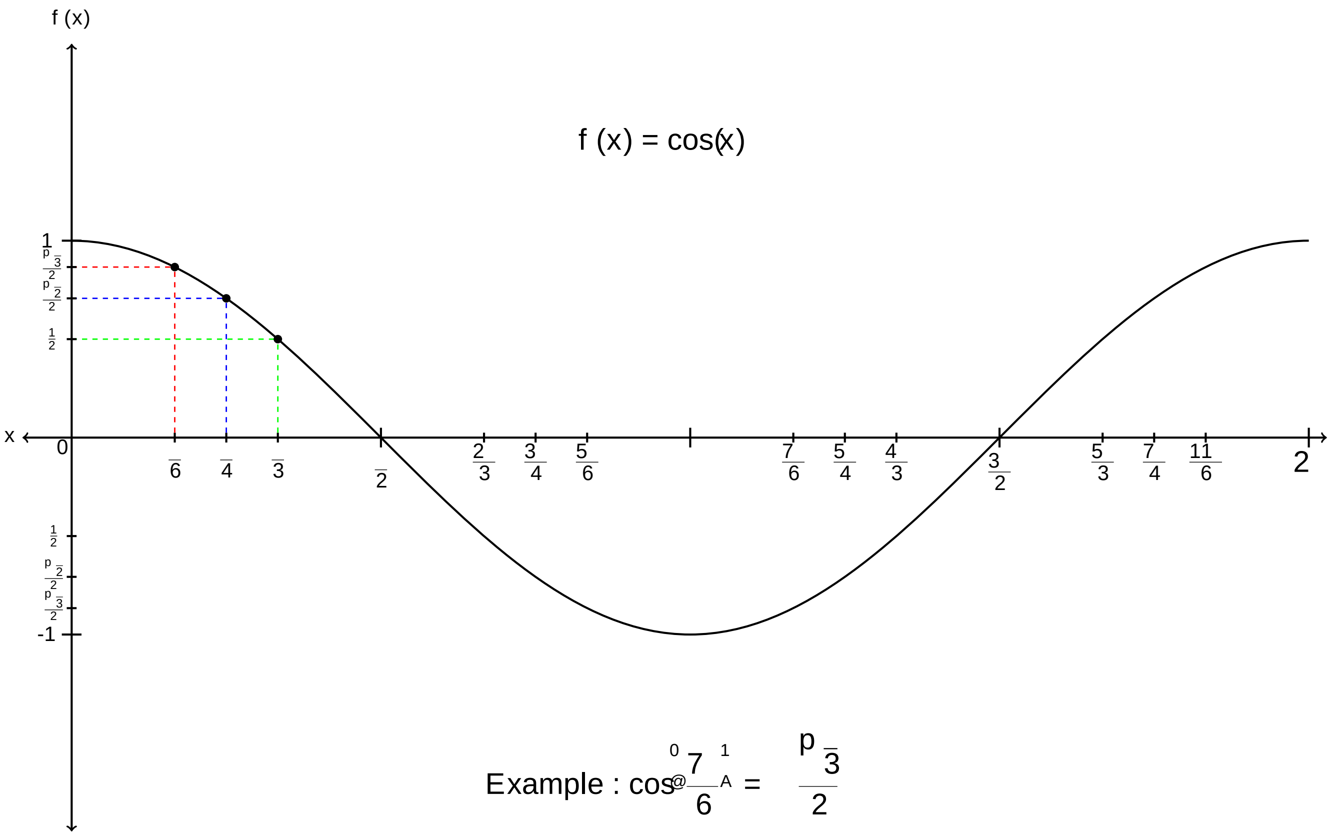
Where $(h;k)$ = vertex

a=scaling factor

6



Example : $\sin\left(\frac{5}{4}\right) = \frac{\sqrt{2}}{2}$



Example : $\cos\left(\frac{7}{6}\right) = \frac{\sqrt{3}}{2}$